

## Valuing Individual Patents Comprising a Portfolio

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Part Two of a Two-Part Series

The first installment of this series described the valuation of a patent monopoly secured by one or more related patents. This installment presents a methodology for valuation of individual patents comprising a multi-patent portfolio. Such analysis is necessary for the appraisal of a patent subject to sale, purchase, license or donation. It may also be useful for managerial decisions related to payment of patent maintenance fees or abandonment of certain patents.

Formulas obtained in the first article equally well describe the value of a single patent or an entire patent portfolio protecting a patented invention. For the purposes of this article, a patent portfolio represents a group of patents protecting a single revenue stream. This stream may be generated by a single product or service, a product line or by the enterprise as a whole (savings realized by using a patented process are treated as imputed revenue). Note that, theoretically, the value of a patent portfolio protecting a single product or service does not depend on the number of patents in the portfolio, as long as they do not extend the scope of the patent monopoly.

As shown in the previous article, the present value of a patent portfolio is calculated using the following general formula:

$$1. PV(PP) = \sum_{i=1}^l \frac{D_i}{(1+I_i)^i}$$

where  $PV(PP)$  is the present value of the patent portfolio  $PP$ ;  $\Delta_i$  is an incremental value of the patent monopoly in year  $i$ ;  $I_i$  is the discount interest rate in year  $i$ ; and  $l$  is the term of the patent monopoly determined by the remaining life of the subsisting patents in the portfolio.  $\Delta_i$  is defined as the incremental profit resulting from the patent monopoly:

$\Delta_i = \overline{PRFT}_i - PRFT_i$ , where  $\overline{PRFT}_i$  is the profit obtained in year  $i$  under the conditions of patent monopoly, and  $PRFT_i$  is the profit in the same year  $i$  in a hypothetical freely-competitive environment without the benefit of patent protection. Assuming for simplicity that the fixed costs in both scenarios are the same and, therefore, the incremental profit can be represented by the incremental gross profit, this formula can be further delineated as:

$$\Delta_i = (\overline{PR}_i - \overline{CG}_i) \times \overline{S}_i - (PR_i - CG_i) \times S_i \text{ where } PR_i$$

is the price of goods sold in year  $i$ ;  $\overline{CG}_i$  is the cost of goods sold in year  $i$ ;  $\overline{S}_i$  is the number of units sold in year  $i$  (all forecasted under conditions of patent monopoly); and  $PR_i$ ,  $CG_i$  and  $S_i$  are, respectively, the price, cost of goods, and units of the same goods in the same year  $i$ , but forecasted under the freely-competitive conditions without regard to the patent monopoly.

Suppose there are  $n$  patents in a portfolio. It would be logical to assume that the value of any constituent patent in this portfolio is its *pro rata* share in the value

of the portfolio as a whole:

$$2. V(P) = \frac{1}{n} \times V(PP)$$

However, this is true only when all patents in the portfolio are coterminous and are of relatively equal strength and scope. We intend to consider the more general case of a portfolio of patents with different terms, scope and relative strength. To do that, the analysis must be performed on an annual basis.

Previously, we introduced the concept of a Patent Portfolio Weight Matrix  $\mathbf{P}$  (see A. Poltorak and P. Lerner, *Essentials of Intellectual Property*, Wiley, NY, 2002, pp 219-224). Essentially, this is a table wherein the rows correspond to the years of the portfolio's life, and the columns correspond to the individual patents in the portfolio. An active patent  $j$  in year  $i$  is represented by the matrix element  $p_j^i$ , which is a positive number ( $0 < p_j^i \leq 1$ ). If a certain patent is not "active" in a given year,  $i$  is either not yet issued or already expired, the corresponding matrix element is set to zero. The matrix elements must satisfy a simple rule: the sum of all elements in any row of the matrix must be equal to one:

$$3. \sum_{j=1}^n p_j^i = 1$$

The meaning of this rule is that, no matter how many patents protect an invention, at the end of the day one can only get one monopoly on the invention.

For example, let us consider a patent portfolio consisting of four patents over a period of 5 years. Suppose, during the first year, the portfolio consisted of only one patent —  $P_1$ ; during the second year, it consisted of two patents —  $P_1$

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and  $P_2$  (the second patent just issued); during the third year, there is only one patent —  $P_2$  (the first patent expired at the end of the second year); during the fourth year, there are two patents —  $P_2$  and  $P_3$ ; and during the fifth year, there are three patents —  $P_2$ ,  $P_3$  and  $P_4$ . Let us present these facts as a table:

	Patent 1	Patent 2	Patent 3	Patent 4
Year 1	1	0	0	0
Year 2	0.5	0.5	0	0
Year 3	0	1	0	0
Year 4	0	0.5	0.5	0
Year 5		0.33	0.33	0.33

At this point, we have apportioned the values *pro rata* according to the number of patents active that year, *ie* the value in a given cell was chosen as  $1/n$ , where  $n$  is the number of patents active that year.

The Patent Portfolio Weight Matrix **P** looks, in this case, as follows:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0.33 & 0.33 & 0.33 \end{pmatrix}$$

The Patent Portfolio Weight Matrix **P** gives a complete picture of which patents are active in any given year over the life of the portfolio.

To account for the possibility that some of the patents may not have been active during the entire year, we should assign to a patent a number weighted according to the number of months the patent is active that year. Suppose that the third patent issued in the beginning of July. Then, instead of 0.5, the value for  $p_3$  is 0.25. This automatically raises the value of the second patent in the third row (because the sum of all elements in the row must be equal to 1):

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.75 & 0.25 & 0 \\ 0 & 0.33 & 0.33 & 0.33 \end{pmatrix}$$

Aside from different terms, patents may have different values based on their relative strength and scope. A broad patent on a basic technology is not equal in value to a narrow patent on a minor improvement. If the first patent in our example was such a broad patent, responsible for, let's say, 80% of the portfolio value that year, instead of 0.5 we would assign to it the value 0.8, leaving 0.2 for the second "improvement" patent:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.75 & 0.25 & 0 \\ 0 & 0.33 & 0.33 & 0.33 \end{pmatrix}$$

Although, as mentioned before, strictly speaking, the value of a portfolio does not depend on the number of constituent patents (so long as they protect the same monopoly), the portfolio is still a sum of its constituent patents and, therefore, the annual value of a portfolio  $PP$  is the sum of the annual values of the individual patents:

$$4. AV_i(PP) = \sum_{j=1}^n AV_i(P^j)$$

where  $AV_i(PP)$  is the annual value of the portfolio in year  $i$ ;  $AV_i(P^j)$  is the annual value of the  $j$ -th patent,  $P_j$ , in year  $i$ ; and  $n$  is the number of patents in the portfolio.

Since, according to 0, the sum of all matrix elements in one row is equal to 1, we can multiply the left side of the equation by such a sum without changing the equation:

$$5. AV_i(PP) \times \sum_{j=1}^n p_i^j = \sum_{j=1}^n AV_i(P^j)$$

Since the annual value of the portfolio,  $AV_i(PP)$ , is a constant, we can rewrite this equation as:

$$6. \sum_{j=1}^n AV_i(PP) \times p_i^j = \sum_{j=1}^n AV_i(P^j)$$

From here it is logical to assume (although, strictly speaking, it is not the only possible solution of equation 0) that additive members on both sides of

the equation are equal:

$$7. AV_i(PP) \times p_i^j = AV_i(P^j)$$

Since the annual value of the patent portfolio is, by definition, the annual value of the patent monopoly, we have:

$$8. AV_i(P^j) = p_i^j \times D_i$$

Once we know the annual values of the patent  $P^j$ , it is easy to discount them to the present value:

$$9. PV(P^j) = \sum_{i=1}^I \frac{p_i^j \times D_i}{(1+I_i)^i}$$

This expression allows one to calculate the present value  $PV(P^j)$  of a constituent patent  $P^j$  based on the Patent Portfolio Matrix  $P=(p_i^j)$  and the annual values of the patent monopoly. Expression (9) for the present value of a constituent patent differs from expression (1) for the present value of the portfolio to the extent that, in expression (9), the annual value of the patent monopoly is weighted for the relative contribution of a constituent patent to the overall value of the portfolio.

The analysis presented here, as well as that presented in the first article, addresses valuation of an individual patent or a patent portfolio in an ideal environment free of patent infringement. In the future we will consider a more realistic scenario wherein a patent value is adjusted for the risks associated with patent enforcement.

